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This expression is negative for $n>2$ and therefore decreases as n increases. Hence the area increases as n increases. Q. E. D.

NOTE. Equating the derivative to zero, we have

$$\tan\frac{2\pi}{n}-\frac{2\pi}{n}=0.$$

This equation is true only for very small values of the angle. In fact, it approximates to exactitude as the angle $2\pi/n$ approaches zero. Hence, the area of the polygon opproaches its maximum value (as a limit) as n approaches ∞; it reaches its maximum when $n=\infty$. In that case the polygon becomes a circle, which is, therefore, the greatest of all isoperimetric plane figures. In a similar manner, it may be shown that of all plane figures of given area the circle has the least periphery.

Also solved by J. E. Sanders, V. M. Spunar, J. Scheffer, and S. G. Barton.

299. Proposed by JOSEPH V. COLLINS, Ph. D., State Normal School, Stevens Point, Wisconsin.

A cow is pasturing outside a circular field containing 10 acres. What length of rope will allow her to graze over exactly two acres?

Solution by J. SCHEFFER, A. M., Hagerstown, Md.; J. E. SANDERS, Columbus, Ohio, and S. G. BARTON, Ph. D., Clarkson School of Technology.

It is assumed that the cow is tethered at a point in the circumference of the field.

It is evident that the area of the pasture consists of a semi-circle, of radius equal to the length of the rope and two portions contained between the circular boundary of the field and the involute of the circle.

Let a be the radius of the field, l the length of the rope. The intrinsic equation of the involute of a circle is

$$s=a\frac{\psi^2}{2}.$$

Considering the element of area contained between this involute and its evolute, that is, the circle, and two radii of curvature, we have

$$dA=\rho\frac{ds}{2},\quad \rho=\frac{ds}{d\psi}=a\psi;\quad \therefore s=\frac{\rho^2}{2a},\quad ds=\frac{\rho}{a}d\rho.\quad \therefore dA=\frac{\rho^2}{2a}d\rho.$$

The limits of ρ for area involved are evidently 0 and l.

$$\therefore A=\int_0^l\frac{\rho^2}{2a}d\rho=\frac{l^3}{6a}.\quad \text{Hence area}=2\text{ acres}=\frac{\pi l^2}{2}+\frac{l^3}{3a}.$$

Using data of problem, $a=372.4$ feet. $.0009l^3+1.5708l^2=87120$. $l=221.9$ feet.

Solved in a slightly different way by V. M. Spunar.

NUMBER THEORY AND DIOPHANTINE ANALYSIS.

Edited by Dr. G. E. Wahlin, University of Illinois.

175. Proposed by H. C. FEEMSTER, A. B., Professor of Mathematics, York College, York, Nebraska.

Show (a) that $[(2n)!]/[(n+1)!n!]$ is an integer, and (b) that $[(2a)!(2b)!]/[a!b!(a+b)!]$ is an integer.

Solution by G. E. WAHLIN, Ph. D., University of Illinois.

(a) To show that $\frac{(2n)!}{(n+1)!(n!)^2}$ is an integer.

$$\frac{(2n)!}{(n+1)\ (n!)^2}=\frac{(2n)!}{(n+1)!\,n!}=\frac{2n(2n-1)\ (2n-2)\ldots(n+1)}{(n+1)!}.$$

$I=\frac{2n\ (2n-1)\ldots(n+1)}{n!}$ is an integer, being the quotient of n successive integers by $n!$.

For the same reason, $I'=\frac{(2n+1)\ (2n)\ (2n-1)\ldots(n+1)}{(n+1)!}$ is an integer.

But $I'=\frac{2n+1}{n+1}I$; and since $2(n+1)-(2n+1)=1$,

the numerator and denominator are relatively prime in $\frac{2n+1}{n+1}$, and therefore since $I.\frac{2n+1}{n+1}$ is an integer, I must be divisible by $n+1$, or $\frac{I}{n+1}$ is an integer.

But $\frac{I}{n+1}=\frac{2n\ (2n-1)\ldots(n+1)}{(n+1)!}=\frac{2n!}{(n+1)\ (n\,!)^2}$.

(b) To show that $\frac{(2a)!(2b)!}{a!b!\ (a+b)!}$ is an integer.

It is evident that the given expression is equal to

$$\frac{(a+1)\,(a+2)\ldots.2a(b+1)\,(b+2)\ldots.2b}{(a+b)!}\ldots(1),$$